Abstract:

A model of heat and mass transfer for a spherical surface under conditions of natural convection in drying processes is considered. The condition of equality of dynamic, thermal, concentration boundary layers is accepted. Criteria dependences for determining the average Nusselt (heat transfer) and Sherwood (mass transfer) numbers are given.

Keywords: Drying objects, apricots, plums, grapes, spherical body, heat and mass transfer, dynamic layer, heat flow, similarity theory.

Introduction

A model of heat and mass transfer for a spherical surface under conditions of natural convection in drying processes is considered. The condition of equality of dynamic, thermal, concentration boundary layers is accepted. Criteria dependences are given for determining the average Nusselt (heat transfer) and Sherwood (mass transfer) numbers.

Such objects of drying, as apricots, plums, grapes, etc., can be considered as spherical bodies, and the general mechanism of heat and mass transfer - as the transfer of heat and
mass for a spherical surface [1, 1992].

The Main Findings And Results

In the theory of similarity for the processes of heat and mass transfer in gases, the equality of the thicknesses of the dynamic, thermal and concentration boundary layers is assumed [2, 1994; 3, 1983].

\[ \delta = \delta_f = \delta_c \]  
(1)

The relationship between the thicknesses of the layers is determined only by the physical properties of the medium, namely those that characterize its reaction to the passage of flows of heat, steam and the amount of motion. The thickness of the dynamic layer, according to [2, 1994].

\[ \delta = 4.64 x \sqrt{Re_x} \]  
(2)

For natural convection conditions, the Reynolds number is replaced by the Archimedes number [4, 1986]

\[ Re_x = \sqrt{Ar_x} ; \quad Ar_x = Gr_x + Gr_{mx} \]  
(3)

\[ \delta = 4.64 \cdot x / \sqrt{Ar_x} \]  
(4)

where \( Gr, Gr_x \) – are heat and mass transfer Grashof numbers. Thermal boundary layer thickness [2, 1994].

\[ \delta_T = 4.64 / \sqrt{Pr} \cdot x / \sqrt{Gr_x} \]  
(5)

(Pr - Prandtl number).

The ratio of the thicknesses of the thermal and dynamic layers

\[ \delta_T / \delta = 1 / \sqrt{Pr} . \]  
(6)

For air \( Pr = 0.71 \); \quad \delta_T / \delta = 1.12 .

The thickness of the concentrated layer is determined in a similar way [2, 1994; 3, 1983]

\[ \delta_c = 4.64 / \sqrt{Sc} \cdot x / \sqrt{Gr_{mx}} \]  
(8)

\( Sc \) – the Schmidt number.

The ratio of the concentration and dynamic layers

\[ \delta_T / \delta = 1 / \sqrt{Sc} \]  
(9)

For air \( Sc = 0.614 \); \quad \delta_c / \delta = 1.176

(10)
From relations (6) and (9)

$$\frac{\delta_T}{\delta_c} = \sqrt[3]{ \frac{Sc}{Pr} }$$

(11)

Hence,

$$\frac{\delta_T}{\delta_c} = 0.953$$

(12)

Relations (7), (10) and (12) show that

- the thickness of the thermal and concentrated layers is greater than the dynamic

$$\delta \leq \delta_T \leq \delta_c$$

(13)

- condition (1) is acceptable.

Condition (1) is important, since in this case the heat, mass transfer, and dynamic

problems become identical. An integral method with series expansion is used for the solution

[3, 1983]. The integral equation for a two-dimensional axisymmetric flow will take the form (figure):

- for heat exchange

$$\frac{1}{r} \frac{d}{dr} \left( r \int_{0}^{\delta} U_x^2 dy \right) = g \beta t \sin \gamma \left( \int_{0}^{\delta} \Theta dy \right) - \nu \left( \frac{dU_x}{dy} \right)_{y=0}$$

(14)

$$\frac{1}{r} \frac{d}{dx} \left( r \int_{0}^{\delta} U_x \Theta dy \right) = -a \left( \frac{d\Theta}{dy} \right)_{y=0}$$

(15)

$$\Delta t = t_M - t_B$$

$$\Theta = \frac{t - t_B}{t_M - t_B}$$

(16)

- for mass transfer

$$\frac{1}{r} \frac{d}{dr} \left( r \int_{0}^{\delta} U_x^2 dy \right) = g \beta c \sin \gamma \left( \int_{0}^{\delta} \Theta_c dy \right) - \nu \left( \frac{dU_x}{dy} \right)_{y=0}$$

(17)

$$\frac{1}{r} \frac{d}{dx} \left( r \int_{0}^{\delta} U_x \Theta dy \right) = -a_c \left( \frac{d\Theta_c}{dy} \right)_{y=0}$$

(18)

$$\Delta c = c_M - c_B$$

$$\Theta_c = \frac{c - c_B}{c_M - c_B}$$

(19)

Where $r$ – is the current radius, the distance from the vertical axis of the sphere to

the point under consideration, m; $R$ – radius of the sphere, m; $x, y$ – coordinates along and

normal to the surface of the sphere, m; $\gamma$ – the angle between the tangent to the surface of
the sphere and the horizontal, \( \text{grad} \); \( U_x \) – air speed, \( \text{m} / \text{s} \); \( g \) – acceleration of gravity, \( \text{m} / \text{s}^2 \); \( \beta, \beta_c \) – thermal and concentration coefficients of density change; \( T_m, T_b \) – surface and air temperatures, \( ^0\text{C} \); \( c_m, c_B \) – vapor concentration on the surface and in the air; \( \nu, a, a_c \) – coefficients of kinematic viscosity, conductivity and vapor diffusion temperatures, \( \text{m}^2 / \text{s} \).

As a first approximation at \( y = 0 \) : \( \Theta = 1, \Theta_c = 1, U_x = 0 \); at \( y = \delta_c \); \( \Theta = \Theta_c = U_x = \frac{\partial \Theta}{\partial y} = \frac{\partial \Theta_c}{\partial y} = \frac{\partial U_x}{\partial y} = 0 \).

Profiles are described by the following expressions:

\[
\Theta = \left(1 - \frac{y}{\delta_c}\right)^2, \quad \Theta_c = \left(1 - \frac{y}{\delta_c}\right)^2, \quad U_x = U(x) \left(1 - \frac{y}{\delta}\right)^2,
\]

(20)

where \( U(x) \) – is the average air flow velocity in the boundary layer, \( \text{m} / \text{s} \).

The lower critical point is located at \( x = 0 \). Then \( \gamma = 2x/D \), where \( D \) – is the diameter.

Expanding the sought functions and \( \sin \gamma \) in series of \( x/D \), we calculate the sums of the series for a large number of terms. Nusselt average

\[
\bar{N}u = \frac{2}{\pi} \int_0^{\pi} \bar{N}u \left(\frac{x}{D}\right) d\left(\frac{x}{D}\right) = b(Gr \text{Pr})^{1/4}
\]

(21)

At \( \text{Pr} = 0.7 \)

\[
\bar{N}u = 0.474(Gr \text{Pr})^{1/4}
\]

(22)

Likewise, Sherwood's average

\[
\bar{S}h = \frac{2}{\pi} \int_0^{\pi} \bar{S}h \left(\frac{x}{D}\right) d\left(\frac{x}{D}\right) = b(Gr_M \text{Sc})^{1/4}
\]

(23)

At \( \text{Sc} = 0.614 \)

\[
\bar{S}h = 0.41(Gr \text{Sc})^{1/4}
\]
Of practical interest are the relations obtained empirically. In [3, 1983; 5, 1983; 6, 1982; 7, 2017], the most frequently used expressions for spherical bodies under natural convection are given. For heat transfer

\[
\overline{Nu} = 2 + 0.43Ra^{1/3}, \quad 1 < Gr < 10^5
\]

(25)

\[
\overline{Nu} = 2 + 0.45Ra^{1/2}, \quad 1 < Gr
\]

(26)

\[
\overline{Nu} = 2 + 0.46Ra^{1/4}, \quad 1 < Ra < 10^5
\]

(27)

\( Ra = GrPr \) – Rayleigh number). For mass transfer

\[
\overline{Sh} = 2 + 0.59(Gr_MSc)^{0.25}, \quad Gr_MSc < 10^5
\]

(28)

The constant 2 in (25) - (28) appears in view of the fact that at \( Gr \) and \( Gr_M \to 0 \), \( \overline{Nu} \) and \( \overline{Sh} \to 2 \), which corresponds to the case of pure thermal conductivity and vapor diffusion through a layer of stationary air surrounding the sphere [8, 2013; 9, 2015; 10, 2015].

**Conclusion**

From the above expressions, it can be seen that the value of the Nusselt and Sherwood criteria plays an important role in solving problems related to heat-mass transfer processes in a spherical portable solar dryer. A high value of the Nusselt criterion in turn leads to a large value of the heat transfer coefficient, as well as an acceleration of the heat transfer process, a high Sherwood criterion leads to an acceleration of the mass transfer process under natural convection.

**References**
