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Bisingular Integral In The Space Of Summable Functions

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ABSTRACT

It is obtained a Zigmund type estimate for the bisingular integral in the space of Summation functions. It is constructed an invariant functional space based on the inequality. Using the method of successive approximations it is proven the solvability of the nonlinear bisingular integral equation in invariant space.

KEYWORDS

Bisingular integral operator, Zigmund type estimate, invariant space, summable function.

INTRODUCTION

The classical boundedness theorem of singular operator with the Hilbert kernel in space L_p ($p > 1$), it was proved by N.H. Luzin in [6] and M.Riesz in [16] for the cases $p=2$ and $p > 1$, respectively.

Subsequently, this result was carried over in a number of papers for fairly wide classes of Jordan rectifiable curves. A detailed prehistory

of this issue is available in the work [9], also in the works of A.P. Calderon [11], [12], and [13].

To study the special integral

$(-\infty < a < b < +\infty)$ with the summable density in the work [4], [10] for a function $u \in L_p^{loc}(a, b)$, where $L_p^{loc}(a, b)$ - is the set of functions, summable with the

degree p in any compact segment of the interval (a, b) . The characteristics were introduced

and in the case $-\infty < a < +\infty$ it is established estimates $\int_a^b |u(x)|^p dx$ by (u) , $[1] \int_a^b |u(x)|^p dx$.

In the limiting case $f = \infty$ and $u \in C[a, b]$ these results were obtained in [3], [7], it was shown that estimates [2] in a certain sense are unimprovable. In [5] using M. Riesz's theorem about the bounded action

for a.e. at u in the case $L_p(a, b)$ the results are obtained in [1], [2].

One of the first papers, dedicated to the repeated special integral with the Hilbert kernel

was a work of L. Cesari [14]. He proved that if

then

Following L. Cesari, I.E. Zak [5] in his work also showed that the class of functions $H_{\delta}^{\alpha}(\delta^{2\alpha})^{1/2}$ is that with

respect to the operator B . In the paper, it was proved that the classes of functions

are invariant with respect to the operator
 B.

RESULTS

Consider a bisingular integral of the form

where the function , and .

For the we introduce the characteristic

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$, i = 1, 2.$

Teopema 1. Let Then from convergence of correspondence integrals it holds the following inequality

where the statistic is defined by

$$p.$$

Moreover, it is obtained estimates u

$$) \quad u) , \quad u) .$$
 We denote by
 the set of site functions $\varphi_{\xi \eta}$

$$(\xi)_{2\eta^2} \quad \psi) \quad (\delta_{-1}, \xi_{-1}, \eta_{-1}$$

$$(\xi)_{2\eta^2} \psi(\xi_{-1}, \eta_{-1} \quad (\delta_{-2}, \xi_{-2})_{2\eta^2}$$

$$) \psi \delta_{\xi \eta} \quad (\delta_{-2}, \xi_{-2})_{2\eta^2}$$

 defined $f \quad \delta_{i \xi i}$

$$\eta_{i>0} \xi_i) + \eta_{i=1} i=b_{-i} a_{-i} \quad i=1,2,$$
 and such

increasing in δ_1, δ_2 (uniformly by other variables), the functions

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CONCLUSION

Using the method of successive approximations it is proven the solvability

of the nonlinear bisingular integral equation

in Ω , where the function is defined on

and λ is a real parameter.

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